Vortices, Q-balls and Domain Walls on Dielectric M2-branes

Vortices, Q-balls and Domain Walls on Dielectric M2-branes

Masato Arai,[a] **Claus Montonen**[b] **and Shin Sasaki**[b]

[a]*Center for Quantum Spacetime (CQUeST), Sogang University,*
Shinsu-dong 1, Mapo-gu, Seoul 121-742, Korea

[b]*Department of Physics, University of Helsinki, Helsinki Institute of Physics,*
P.O.Box 64, FIN-00014, Finland

E-mail: arai@sogang.ac.kr, claus.montonen@helsinki.fi,
shin.sasaki@helsinki.fi

ABSTRACT: We study BPS solitons in $\mathcal{N} = 6$ U(N) × U(N) Chern-Simons-matter theory deformed by an F-term mass. The F-term mass generically breaks $\mathcal{N} = 6$ supersymmetry down to $\mathcal{N} = 2$. At vacua, M2-branes are polarized into a fuzzy S^3 forming a spherical M5-brane with topology $\mathbf{R}^{1,2} \times S^3$. The polarization is interpreted as Myers' dielectric effect caused by an anti-self-dual 4-form flux T_4 in the eleven-dimensional supergravity. Assuming a polarized M2-brane configuration, the model effectively reduces to the well-known abelian Chern-Simons-Higgs model studied in detail by Jackiw-Lee-Weinberg. We find that the potential for the fuzzy S^3 radius agrees with the one calculated from the M5-brane point of view at large N. This effective model admits not only BPS topological vortex and domain wall solutions but also non-topological solitons that keep 1/4 of the manifest $\mathcal{N} = 2$ supersymmetry. We also comment on the reduction of our configuration to ten dimensions.

Contents

1 Introduction

No one doubts that understanding various aspects of low-energy effective theory of M2-branes enables us to uncover mysterious properties of M-theory and non-perturbative physics of string theories. Recently, Bagger, Lambert and Gustavsson proposed a three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons model based a novel 3-algebra (BLG model) [1, 2]. This model has been expected to describe the low energy effective theory of two coincident M2-branes in eleven dimensions. However, it seemed difficult to generalize their model to the case of arbitrary number of M2-branes.

Meanwhile, Aharony-Bergman-Jafferis-Maldacena proposed a three-dimensional $\mathcal{N} = 6$ U(N) × U(N) Chern-Simons-matter theory with level $(k, -k)$ (ABJM model) which may be regarded as the low-energy effective theory of N coincident M2-branes probing $\mathbf{C}^4/\mathbf{Z}_k$ orbifold [3]. The model consists of gauge fields $A_\mu, \hat{A}_\mu$, four complex scalars Z^A, W_A ($A = 1, 2$) in the (anti) bi-fundamental representation of the gauge group, and their superpartners. The model is believed to be a dual description of M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$.

After the proposal by Aharony-Bergman-Jafferis-Maldacena, a lot of works on the effective theory of interacting multiple membranes have been studied. Among them, classical

solutions in the three-dimensional M2-brane world-volume theory attracted much attention since these solutions can be interpreted as various configurations of branes existing in the eleven-dimensional M-theory. In [4], a BPS fuzzy funnel configuration that represents an M5-brane intersecting with multiple M2-branes was found.[1] A domain wall solution that interpolates between fuzzy S^3 and trivial vacua was found [6] in the ABJM model with the D-term mass deformation. On the other hand, time evolutions of fuzzy S^3 and M5/anti-M5 configurations that are generically non-BPS were investigated in [7]. These results provide some evidence that the ABJM model gives a correct description of the dynamics of multiple membranes.

On the other hand, in [8], $AdS_4 \times S^7$ dual theory of M2-branes with reduced supersymmetry was investigated following the prescription studied by Polchinski-Strassler [9]. The author of [8] proposed a mass deformation of multiple membrane theory with $\mathcal{N} = 2$ world-volume supersymmetry and studied a probe M5-brane in the presence of anti-self-dual 4-form flux T_4 in the eleven-dimensional supergravity. The vacua are spherical M5-branes with topology $\mathbf{R}^{1,2} \times S^3$ sharing three dimensions with M2-branes. He conjectured that there exists BPS domain walls that interpolate between various vacua in the model. However, the correct description of the mass deformed multiple M2-brane theory was not known at that time. Note that the corresponding supergravity solutions were analyzed in [10, 11].

In this paper, we study the ABJM model deformed by an F-term mass which was first introduced in [12]. The F-term mass generically breaks $\mathcal{N} = 6$ supersymmetry down to $\mathcal{N} = 2$ preserving $\mathrm{SU}(2)_{\mathrm{diag}}$ global symmetry of $\mathrm{SU}(2) \times \mathrm{SU}(2)$ and the resulting model is identified with the model studied in [8]. We find that a vacuum configuration of the model is a fuzzy S^3. We see that the radius of the sphere derived here coincides with the one obtained in [8] at large-N identifying our vacuum as a spherical M5-brane with topology $\mathbf{R}^{1,2} \times S^3$. From the viewpoint of the M2-brane world-volume theory, this is nothing but Myers' dielectric effect [13] caused by eleven-dimensional supergravity flux $T_4 \sim m$. We also see that reducible configurations found in [12] are identified with a set of fuzzy S^3 shells.

In the latter half of this paper, we study various classes of BPS configurations with polarized M2-brane geometry in the mass deformed ABJM model. Assuming a polarized M2-brane configuration, we find that the Hamiltonian effectively reduces to that of the well-known abelian Chern-Simons-Higgs system with sixth-power potential studied in detail by Jackiw-Lee-Weinberg [14]. It has been known that this model exhibits an $\mathcal{N} = 2$ supersymmetry in three dimensions [15] and admits BPS topological vortices and domain walls. In addition, there exists non-topological soliton solutions [16] and a BPS supertube solution [17]. We find that the potential for the S^3 radius in the effective Hamiltonian has the same structure as found in [8]. Since the configurations corresponding to the BPS solutions can exist only in the case of non-zero m, these solutions are supported against collapse by the non-zero background flux T_4 realizing a generalization of the Myers effect.

The organization of this paper is as follows. In section 2, we introduce the ABJM model deformed by the F-term mass. Equations of motion and $\mathcal{N} = 2$ on-shell supersymmetry transformations are derived. In section 3, we study the vacuum structure of

---

[1]A similar analysis in the BLG model has been performed in [5].

the model. We compare the radius of the fuzzy S^3 with the one found in [8] at large-N finding agreement between them. In section 4, assuming an ansatz, we derive the effective Hamiltonian that reduces to the abelian Chern-Simons-Higgs model studied in [14, 18]. The non-abelian property of fields is totally encoded into the "BPS matrices" first constructed in [12]. We then perform the Bogomol'nyi completion in the effective Hamiltonian combining the kinetic and potential terms and derive the BPS equations. The consistency between these BPS equations and the full equations of motion, as well as the number of preserved supersymmetries are discussed. In section 5, parts of the exact and numerical results of the BPS solutions are discussed. Possible interpretations of these solutions in terms of eleven-dimensional M-theory are briefly discussed in this section. Section 6 is devoted to conclusions and discussions. In appendix A, the explicit form of the BPS matrices is presented. The $\mathcal{N} = 2$ superfield formulation of the ABJM model can be found in appendix B. The effective Lagrangian including fermion parts on the polarized M2-branes can be found in appendix C.

2   The ABJM model

The ABJM model [3] is a $(2+1)$-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons-Higgs model of level $(k, -k)$ with $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge symmetry. The model is expected to describe the low energy world-volume theory of N coincident M2-branes probing $\mathbf{C}^4/\mathbf{Z}_k$. We employ the notation and convention of [19] but with a different normalization of the $\mathrm{U}(N)$ gauge generators T^a such that $\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$. The bosonic part of the massless ABJM action is

$$S = S_{\mathrm{kin}} + S_{\mathrm{CS}} + S_{\mathrm{pot}}, \tag{2.1}$$

where each term is given by

$$S_{\mathrm{kin}} = \int d^3x \, \mathrm{Tr} \left[-(D_\mu Z^A)(D^\mu Z^A)^\dagger - (D_\mu W_A)(D^\mu W_A)^\dagger \right], \tag{2.2}$$

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int d^3x \, \mathrm{Tr} \, \epsilon^{\mu\nu\lambda} \left[A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right], \tag{2.3}$$

$$\begin{aligned}
S_{\mathrm{pot}} = &-\frac{4\pi^2}{k^2} \int d^3x \, \mathrm{Tr} \left[(Z^A Z_A^\dagger + W^{\dagger A} W_A)(Z^B Z_B^\dagger - W^{\dagger B} W_B)(Z^C Z_C^\dagger - W^{\dagger C} W_C) \right. \\
&\qquad\qquad\qquad\quad +(Z_A^\dagger Z^A + W_A W^{\dagger A})(Z_B^\dagger Z^B - W_B W^{\dagger B})(Z_C^\dagger Z^C - W_C W^{\dagger C}) \\
&\qquad\qquad\qquad\quad -2Z_A^\dagger(Z^B Z_B^\dagger - W^{\dagger B} W_B)Z^A(Z_C^\dagger Z^C - W_C W^{\dagger C}) \\
&\qquad\qquad\qquad\quad \left. -2W^{\dagger A}(Z_B^\dagger Z^B - W_B W^{\dagger B})W_A(Z^C Z_C^\dagger - W^{\dagger C} W_C) \right] \\
&+\frac{16\pi^2}{k^2} \int d^3x \, \mathrm{Tr} \left[W^{\dagger A} Z_B^\dagger W^{\dagger C} W_A Z^B W_C - W^{\dagger A} Z_B^\dagger W^{\dagger C} W_C Z^B W_A \right. \\
&\qquad\qquad\qquad\quad \left. +Z_A^\dagger W^{\dagger B} Z_C^\dagger Z^A W_B Z^C - Z_A^\dagger W^{\dagger B} Z_C^\dagger Z^C W_B Z^A \right]. \tag{2.4}
\end{aligned}$$

Here A_μ, $\hat{A}_\mu$ are $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge fields, $Z^A, W^{\dagger A}$ ($A = 1, 2$) are complex scalar fields in the $\mathrm{U}(N) \times \mathrm{U}(N)$ bi-fundamental $(\mathbf{N}, \bar{\mathbf{N}})$ representation. The world-volume metric is

$\eta_{\mu\nu} = \text{diag}(-1,1,1)$. The gauge covariant derivative is

$$D_\mu Z^A = \partial_\mu Z^A + i A_\mu Z^A - i Z^A \hat{A}_\mu \,. \tag{2.5}$$

The gauge field strength is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \,, \tag{2.6}$$

and similarly for $\hat{A}_\mu$. The common U(1) charge is fixed to $+1$. The model exhibits a manifest $\text{SU}(2) \times \text{SU}(2) \times \text{U}(1)_R$ global symmetry. Under the each SU(2)s, Z^A, W_A transform independently in the fundamental representation. Apart from this manifest symmetry, there is an $\text{SU}(2)_R$ symmetry under which the fields $Z^1, W^{\dagger 1}$ (and $Z^2, W^{\dagger 2}$) transform as a doublet. It was discussed in [3] that the $\text{SU}(2) \times \text{SU}(2)$ global symmetry is combined with the $\text{SU}(2)_R$ and enhanced to $\text{SU}(4)_R \sim \text{SO}(6)_R$. Therefore, for $k > 2$, the model has $\mathcal{N} = 6$ supersymmetry. We consider a trivial embedding of the world-volume in the space-time. Namely, the world-volume coordinates (x^0, x^1, x^2) are identified with the space-time coordinates (X^0, X^1, X^2). The four complex scalars $Z^A, W^{\dagger A}$ represent the transverse displacement of the M2-branes along eight directions X^I ($I = 3, \ldots, 10$). The orbifolding symmetry $\mathbf{Z}_k$ act as $(Z^A, W^{\dagger A}) \to e^{\frac{2\pi i}{k}}(Z^A, W^{\dagger A})$. The $\mathcal{N} = 2$ superfield formalism of the model can be found in appendix B.

The Gauss' law constraint comes from the equation of motion for the gauge fields,

$$\frac{k}{4\pi} \epsilon^{\rho\mu\nu} F_{\mu\nu} = i \left[Z^A (D^\rho Z^A)^\dagger - (D^\rho Z^A) Z_A^\dagger \right] + i \left[W^{\dagger A}(D^\rho W_A) - (D^\rho W_A)^\dagger W_A \right], \quad (2.7)$$

$$\frac{k}{4\pi} \epsilon^{\rho\mu\nu} \hat{F}_{\mu\nu} = i \left[(D^\rho Z^A)^\dagger Z^A - Z_A^\dagger (D^\rho Z^A) \right] + i \left[(D^\rho W_A) W^{\dagger A} - W_A (D^\rho W_A)^\dagger \right]. \quad (2.8)$$

The Noether current and charge corresponding to the following U(1) gauge transformation (which we call baryonic U(1))

$$(Z^A, W^{\dagger A}) \longrightarrow e^{i\alpha}(Z^A, W^{\dagger A}) \tag{2.9}$$

are derived as

$$j_b^\mu = -i\text{Tr} \left[Z^A D^\mu Z_A^\dagger - Z_A^\dagger D^\mu Z^A - W_A D^\mu W^{\dagger A} + W^{\dagger A} D^\mu W_A \right], \tag{2.10}$$

$$Q_b = i \int d^2x \, \text{Tr} \left[Z^A D_0 Z_A^\dagger - Z_A^\dagger D_0 Z^A - W_A D_0 W^{\dagger A} + W^{\dagger A} D_0 W_A \right]. \tag{2.11}$$

Let us consider massive deformations of the model. There are two kinds of massive deformations: F- and D-term deformations [12].[2] Here we focus on the F-term mass deformation. The superpotential is given by

$$W = W_0 + \delta W \,, \tag{2.12}$$

where

$$W_0 = \frac{1}{4} \left(\frac{8\pi}{k} \right) \epsilon_{AC} \epsilon^{BD} \text{Tr} \left[\mathcal{Z}^A \mathcal{W}_B \mathcal{Z}^C \mathcal{W}_D \right], \tag{2.13}$$

---

[2]See also [20] for massive deformations with reduced supersymmetries.

is the original superpotential in the massless ABJM model [19] and[3]

$$\delta W = m \text{Tr}\left[\mathcal{Z}^A \mathcal{W}_A\right], \qquad m \in \mathbf{R}, \tag{2.14}$$

is a mass term which breaks the $\text{SU}(2) \times \text{SU}(2)$ global symmetry down to $\text{SU}(2)_{\text{diag}}$ and keeps manifest the $\mathcal{N} = 2$ supersymmetry. This is different from the D-term mass deformation which keeps the manifest $\text{SU}(2) \times \text{SU}(2) \times \text{U}(1)_R \times \mathbf{Z}_2$ symmetry and would preserve $\mathcal{N} = 6$ maximal supersymmetry [12]. Here the $\mathbf{Z}_2$ transformation swaps Z^A and $W^{\dagger A}$. It is easy to derive the equation of motion for the scalar fields. The full equation of motion for Z^A is

$$
\begin{aligned}
D_\mu D^\mu Z^A = \left(\frac{4\pi^2}{k^2}\right)\Big[& (Z^B Z_B^\dagger - W^{\dagger B} W_B)^2 Z^A + (Z^C Z_C^\dagger - W^{\dagger C} W_C)(Z^B Z_B^\dagger + W^{\dagger B} W_B) Z^A \\
& + (Z^C Z_C^\dagger + W^{\dagger C} W_C)(Z^B Z_B^\dagger - W^{\dagger B} W_B) Z^A + Z^A (Z_B^\dagger Z^B - W_B W^{\dagger B})^2 \\
& + Z^A (Z_B^\dagger Z^B - W_B W^{\dagger B})(Z_C^\dagger Z^C + W_C W^{\dagger C}) \\
& + Z^A (Z_B^\dagger Z^B + W_B W^{\dagger B})(Z_C^\dagger Z^C - W_C W^{\dagger C}) \\
& - 2(Z^B Z_B^\dagger - W^{\dagger B} W_B) Z^A (Z_C^\dagger Z^C - W_C W^{\dagger C}) \\
& - 2 Z^B (Z_C^\dagger Z^C - W_C W^{\dagger C}) Z_B^\dagger Z^A - 2 Z^A Z_C^\dagger (Z^B Z_B^\dagger - W^{\dagger B} W_B) Z^C \\
& - 2 Z^A W_B (Z^C Z_C^\dagger - W^{\dagger C} W_C) W^{\dagger B} - 2 W^{\dagger C} (Z_B^\dagger Z^B - W_B W^{\dagger B}) W_C Z^A \\
& - 4 W^{\dagger C} W_B Z^A W_C W^{\dagger B} + 4 W^{\dagger C} W_C Z^A W_B W^{\dagger B} - 4 W^{\dagger B} Z_C^\dagger Z^A W_B Z^C \\
& - 4 Z^C W_B Z^A Z_C^\dagger W^{\dagger B} + 4 W^{\dagger B} Z_C^\dagger Z^C W_B Z^A + 4 Z^A W_B Z^C Z_C^\dagger W^{\dagger B} \Big] \\
& + m^2 Z^A - \frac{4\pi m}{k} \epsilon^{AC} \epsilon_{BD}\Big(W^{\dagger B} W_C W^{\dagger D} + Z^B Z_C^\dagger W^{\dagger D} + W^{\dagger B} Z_C^\dagger Z^D + Z^B W_C Z^D\Big).
\end{aligned}
\tag{2.15}
$$

The equation of motion for $W^{\dagger A}$ is obtained by replacing Z^A with $W^{\dagger A}$ in the above expression.

For later convenience, we derive the Hamiltonian of the model. The Chern-Simons part gives a vanishing contribution because it is a topological quantity. However, the gauge fields enter in the Hamiltonian through the covariant derivative. The result is

$$H = \int d^2x \, \text{Tr}\left[|D_0 Z^A|^2 + |D_i Z^A|^2 + |D_0 W_A|^2 + |D_i W_A|^2 + V_{\text{scalar}}\right], \tag{2.16}$$

where the potential part is

$$V_{\text{scalar}} = V_D + V_F, \tag{2.17}$$

$$
\begin{aligned}
V_D = \frac{4\pi^2}{k^2} \text{Tr}\Big[& \left| Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B} \right|^2 \\
& + \left| W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B \right|^2 \Big],
\end{aligned}
\tag{2.18}
$$

$$V_F = \frac{64\pi^2}{k^2} \text{Tr}\left[\left| \frac{km}{8\pi} W_A + \frac{1}{2}\epsilon_{AC}\epsilon^{BD} W_B Z^C W_D \right|^2 + \left| \frac{km}{8\pi} Z^A - \frac{1}{2}\epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D \right|^2\right]. \tag{2.19}$$

---

[3]Note that its "conjugate" part is given by $\delta \bar{W} = -m\text{Tr}[\bar{\mathcal{W}}^A \bar{\mathcal{Z}}_A]$. The minus sign in front of m comes from the extra minus sign of the components in a "conjugated" superfield [19]. See appendix B.

As we mentioned, the model exhibits at least $\mathcal{N} = 2$ manifest supersymmetry. It can be shown that the model is invariant under the following (on-shell) $\mathcal{N} = 2$ supersymmetry,

$$\delta Z^A = \sqrt{2}\epsilon\zeta^A\,, \tag{2.20}$$

$$\delta W_A = \sqrt{2}\epsilon\omega_A, \tag{2.21}$$

$$\delta\zeta^A = \sqrt{2}\epsilon F^A + i\sqrt{2}\bar{\epsilon}(-Z^A\hat{\sigma} + \sigma Z^A) + \sqrt{2}i\gamma^\mu\bar{\epsilon}D_\mu Z^A\,, \tag{2.22}$$

$$\delta\omega_A = \sqrt{2}\epsilon G_A + i\sqrt{2}\bar{\epsilon}(-W_A\sigma + \hat{\sigma}W_A) + \sqrt{2}i\gamma^\mu\bar{\epsilon}D_\mu W_A\,, \tag{2.23}$$

$$\delta A_\mu = -\frac{i}{\sqrt{2}}(\epsilon\gamma_\mu\bar{\chi}) - \frac{i}{\sqrt{2}}(\bar{\epsilon}\gamma_\mu\chi)\,, \tag{2.24}$$

$$\delta\hat{A}_\mu = -\frac{i}{\sqrt{2}}(\epsilon\gamma_\mu\hat{\bar{\chi}}) - \frac{i}{\sqrt{2}}(\bar{\epsilon}\gamma_\mu\hat{\chi})\,, \tag{2.25}$$

where ζ^A, ω_A are fermionic partners, and $F^A, G_A, \sigma^a, \hat{\sigma}^a, \chi, \hat{\chi}$ are auxiliary fields whose equations of motion are given in (B.10)–(B.14) in appendix B. The supersymmetric transformation is achieved by the operator $\delta = \epsilon Q + \bar{\epsilon}\bar{Q}$ where Q_α is a supercharge and ϵ is a two component complex spinor.

3   Vacua as dielectric M2-branes

In this section, we discuss the physical meaning of our F-term deformation and investigate the vacuum structure of the model. For the massless ABJM model, there is a dual description as M-theory on $AdS_4 \times S^7$ at large-N.[4] We can perturb the $AdS_4 \times S^7$ side by turning on a non-trivial supergravity flux which causes additional terms on the dual M2-brane side. Indeed, in [8], Bena considered a mass perturbation of the theory of N coincident M2-branes. Its fermionic part is given by

$$\delta\mathcal{L} = \text{Re}\left(m\sum_{I=1}^{4}\Lambda_I^2\right). \tag{3.1}$$

Here Λ_I are complexified Majorana fermions living on the M2-branes. The mass term keeps at least $\mathcal{N} = 2$ supersymmetry of the $\mathcal{N} = 8$ maximal supersymmetry. The mass terms for the scalar fields break $SO(8)_R$ symmetry down to $SO(4)_R$. We will discuss this issue of global symmetry a little bit in detail later. The author of [8] conjectured that the deformed model is given by a superpotential of the form

$$W \sim z_1 z_2 z_3 z_4 + m\sum_{I=1}^{4} z_I^2\,, \tag{3.2}$$

where $z_a = x_{a+2} + ix_{a+6}, \ (a = 1, \ldots, 4)$ are $\mathcal{N} = 2$ chiral superfields whose lowest components are complex scalars representing fluctuations along transverse directions to the M2-brane world-volume. The non-zero mass parameter on the M2-brane side was identified with non-trivial anti-self-dual constant 4-form flux T_4 on the supergravity side via

---

[4]We here focus on the $k = 1$ sector where the orbifolding by $\mathbf{Z}_k$ becomes trivial. However, we expect that the results in this section hold even for the $k > 1$ case.

AdS/CFT duality. The 4-form flux has a non-trivial value in the directions transverse to the M2-branes. The "(anti) self-dual" means that the flux satisfies

$$\frac{1}{4!}\epsilon_{ijkl}{}^{mnop}T_{mnop} = \pm T_{ijkl}\,, \tag{3.3}$$

where $\epsilon_{ijkl}{}^{mnop}$ is the eight-dimensional epsilon tensor and indices $i, j, \ldots$ run from 3 to 10.

One may imagine that the M2-branes are polarized due to the background flux forming another higher dimensional brane in eleven-dimensions. The most natural candidate of this higher dimensional brane is an M5-brane. Indeed, the vacuum structure of this deformed model was studied through an examination of a probe M5-brane with N-units of M2-brane charge in the $AdS_4 \times S^7$ perturbed by the flux. For the case of four equal masses, which is our concern here, the supersymmetric vacuum is a configuration of an M5-brane with geometry $\mathbf{R}^{1,2} \times S^3$. Its $\mathbf{R}^{1,2}$ directions are shared with the M2-branes while other three-dimensions are wrapped on an S^3 inside the S^7. The radius r of the S^3 was derived by evaluating the M5-brane action via the Pasti-Sorokin-Tonin and Perry-Schwarz approaches [21, 22]. Its "potential" term at large-N is [8]

$$V(z) = \frac{3T_5}{4A}(r^3 - 4rmA/3)^2, \tag{3.4}$$

where $A = 4\pi N/M_{11}^3$ and M_{11} is the eleven-dimensional Planck mass. Here we have added the overall M5-brane tension T_5, which was ignored in [8], and we have restricted ourselves to the 3456 plane in the 3-7, 4-8, 5-9, 6-10 planes using SO(4) rotations [8]. From the result (3.4), we find a supersymmetric vacuum

$$r^2 \sim N\frac{m}{M_{11}^3}\,. \tag{3.5}$$

Apart from the non-zero radius, there is also a trivial vacuum $r = 0$. Therefore there should exist a domain wall solution which interpolates between these vacua. It was conjectured that the tension of a domain wall is

$$\tau_{DW} \sim m^2 N^2. \tag{3.6}$$

The above situation can be extended to the case that the N M2-branes are uniformly distributed on more than one 3-sphere, with M5-brane charges. The potential felt by a probe M5-brane with M2-brane charges is still written by (3.4), but with a different radius r_b and M2-brane charges n_b. Its form is given by (once again restricting to the 3456 plane),

$$V = \sum_b \frac{3T_5}{4A_b}(r_b^3 - 4r_b m A_b/3)^2, \tag{3.7}$$

where $A_b \equiv 4\pi n_b/M_{11}^3$. Therefore the ground states form M2-brane 3-sphere shells with radii $r_b = 4mA_b/3$.

With these results in mind, let us return to our model. The ABJM model deformed by the F-term mass is given by the superpotential (2.12). This is just the superpotential (3.2)

conjectured in [8]. Its fermionic part $\mathrm{Re}(2m\mathrm{Tr}[\zeta^A\omega_A])$ is nothing but the equation (3.1) via the identification

$$\zeta^A = \frac{1}{\sqrt{2}}(\Lambda^A + i\Lambda^{A+2}), \qquad \omega_A = \frac{1}{\sqrt{2}}(\Lambda^A - i\Lambda^{A+2}). \qquad (3.8)$$

The mass perturbation part (2.14) keeps $\mathrm{SU}(2)_{\mathrm{diag}}$ which is a subgroup of the expected R-symmetry group $\mathrm{SO}(4)_R$. However, we remind the fact that for the massless ABJM model, the global $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry does not commute with the $\mathrm{SU}(2)_R$, they combine giving an $\mathrm{SU}(4)_R$ symmetry. This mechanism of symmetry enhancement would be true even for our case. Although it is not manifest here, the remaining $\mathrm{SU}(2)_{\mathrm{diag}}$ global symmetry would be combined with $\mathrm{SU}(2)_R$ generating $\mathrm{SU}(2)_{\mathrm{diag}} \times \mathrm{SU}(2)_R \sim \mathrm{SO}(4)_R$.

The supersymmetric vacuum of our model can be obtained as follows [12]. Assuming a configuration

$$Z^A = W^{\dagger A} = f_0 S^A, \qquad f_0 \in \mathbf{C}, \qquad (3.9)$$

the condition $V_D = 0$ is automatically satisfied. Here S^A are BPS matrices [12] presented in appendix A. The F-term condition

$$\frac{\partial \mathcal{W}}{\partial Z^A} = -\left(\frac{km}{8\pi} - \frac{1}{2}|f_0|^2\right)\bar{f}_0 S_A^\dagger = 0, \qquad \frac{\partial \mathcal{W}}{\partial W_A} = -\left(\frac{km}{8\pi} - \frac{1}{2}|f_0|^2\right)f_0 S^A = 0 \quad (3.10)$$

yields the following vacua

$$A_\mu = \hat{A}_\mu = 0, \qquad |f_0|^2 = 0, \frac{km}{4\pi}, \qquad (3.11)$$

where $f_0 = 0$ is a trivial vacuum. At the vacuum corresponding to $f_0 \neq 0$, the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge symmetry is broken to $\mathrm{U}(1) \times \mathrm{U}(1)$. One of these U(1)s is the "baryonic U(1)", eq. (2.9), and the other one being an independent rotation of a degree of complexity freedom which decouples when the ansatz (3.9) is used. To see the physical meaning of $f_0 \neq 0$ solution, let us define the following combinations of the transverse fluctuation modes

$$U_\pm^A \equiv \frac{1}{2}(Z^A \pm W^{\dagger A}), \qquad (A = 1, 2). \qquad (3.12)$$

Due to the ansatz $Z^A = W^{\dagger A}$, only U_+^A is non-zero. Therefore half of the eight transverse degrees of freedom X^I ($I = 3, \ldots, 10$) is now dropped and this configuration is nothing but a fuzzy S^3 with radius

$$R^2 = \frac{2}{NT_2}\mathrm{Tr}[U_+^A U_{+A}^\dagger] = \frac{1}{T_2}(N-1)\frac{km}{2\pi} \sim N\frac{m}{M_{11}^3} \qquad (N \to \infty). \qquad (3.13)$$

Here T_2 is a tension of an M2-brane. We have used the relation that the p-brane tension in eleven-dimensions is given by $T_p = (2\pi)^{-p}M_{11}^{p+1}$. This result coincides with the equation (3.5) obtained from the dual M5-brane description. Therefore the vacuum configuration (3.11) is interpreted as polarized M2-branes caused by the background flux T_4 with its VEV m. This is just an M-theory realization of Myers' dielectric effect [13].

Because the model admits vacua $f_0 = 0$, $f_0 \neq 0$ and in each sector they have different values of the superpotential, there is a domain wall solution that interpolates between them. Indeed, it is easy to find such a solution with vanishing gauge fields. The Bogomol'nyi completion of the energy density for the configuration $Z^A = Z^A(x_1)$, $W_A = W_A(x_1)$ with vanishing gauge fields is

$$
\begin{aligned}
E &= \int dx^1 \, \mathrm{Tr} \left[\partial_1 Z^A \partial_1 Z_A^\dagger + \partial_1 W_A \partial_1 W^{\dagger A} + V_D + V_F \right] \\
&= \int dx^1 \, \mathrm{Tr} \left[\left| \partial_1 Z^A - m W^{\dagger A} + \frac{4\pi}{k} \epsilon^{AC} \epsilon_{BD} W^{\dagger B} Z_C^\dagger W^{\dagger D} \right|^2 \right. \\
&\qquad \left. + \left| \partial_1 W^{\dagger A} - m Z^A + \frac{4\pi}{k} \epsilon^{AB} \epsilon_{CD} Z^C W_B Z^D \right|^2 + V_D \right] + T, \quad (3.14)
\end{aligned}
$$

where the surface term is

$$
\begin{aligned}
T = - \int dx^1 \, \partial_1 \mathrm{Tr} \left[-m Z^A W_A - \frac{2\pi}{k} \epsilon_{AC} \epsilon^{BD} Z^A W_B Z^C W_D \right. \\
\left. -m W^{\dagger A} Z_A^\dagger + \frac{2\pi}{k} \epsilon^{AC} \epsilon_{BD} Z_A^\dagger W^{\dagger B} Z_C^\dagger W^{\dagger D} \right]. \quad (3.15)
\end{aligned}
$$

The BPS equations are obtained by requiring that $V_D = 0$ and the vanishing condition inside the absolute values in the equation (3.14). A solution to these equations is given by

$$
Z^A = W^{\dagger A} = f(x_1) S^A, \qquad f^2(x_1) = \frac{k}{4\pi} \frac{m}{1 + e^{-2mx_1}}, \quad (3.16)
$$

which also satisfies the Gauss' law (2.7) and (2.8). This solution is just the one first found in [6], but they obtained it in the D-term deformed theory.

We can see that the solution (3.16) keeps a half of $\mathcal{N} = 2$ supersymmetry specified by

$$
\mathcal{Q}_\pm \equiv Q_1 \pm i \bar{Q}_2, \quad (3.17)
$$

while the one in [6] would keep half of the maximal $\mathcal{N} = 6$ supersymmetry. The tension of the domain wall is evaluated as

$$
T = N(N-1) \frac{km^2}{4\pi} \sim m^2 N^2, \qquad (N \to \infty). \quad (3.18)
$$

This agrees with the result (3.6).

Before going to the next section, we would like to consider the reducible vacuum solutions discussed in [12]. Because the BPS matrices S^A for any partition of N do satisfy the vacuum condition, there is a set of reducible solutions that is obtained by replacing S^A by

$$
\tilde{S}^A = \begin{pmatrix} S_{N_1}^A & & \\ & \cdots & \\ & & S_{N_l}^A \end{pmatrix}. \quad (3.19)
$$

Here N_b, $(b = 1, \ldots, l)$ satisfy $\sum_{b=1}^l N_b = N$ and $S_{N_b}^A$ are the BPS matrices in an N_b dimensional representation. We will see in a later section that these vacua correspond to a set of fuzzy S^3 shells with different values of the radius and that the potential is written as in (3.7).
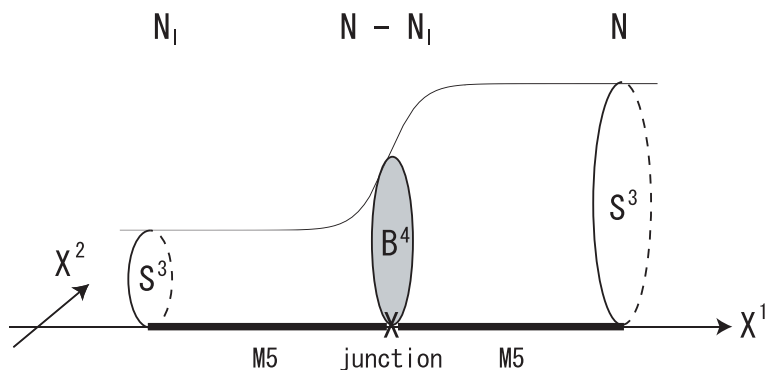
– 9 –

Figure 1. A schematic picture of a domain wall solution.

It is possible to capture the brane configuration corresponding to a domain wall interpolating between one of the reducible vacua constructed, for example, by $\tilde{S}^A = \text{diag}(S^A_{N_l}, 0_{N-N_l})$, $N_l < N$ at $x_1 = -\infty$ and the irreducible vacuum with dimension N at $x_1 = +\infty$. It is easy to show that the large-N behavior of its tension is the same as in (3.18). Following the discussion in [8, 9], in the region $x_1 < 0$, the configuration is interpreted as an M5-brane extending in the (x^0, x^1, x^2) directions and wrapping an S^3 with M2-brane charge N_l while in the region $x_1 > 0$, it is an M5-brane extending along the same directions and wrapping an S^3 with charge N. They are generically both bent and meet at a point $x_1 = 0$. Because of charge conservation, another new M5-brane with charge $N - N_l$ should exist at that point. This is a kind of brane junction. Because all M5-branes meet at a point, one direction should be supplemented for the new M5-brane. This is possible if we consider a ball B^4 instead of a sphere S^3. Therefore the other new M5-brane is extending along the (x^0, x^2) directions and filling a four-ball B^4 with its surface S^3. A schematic picture is given in figure 1.

4 Effective Hamiltonian and BPS equations

In this section, motivated by the discussion in the previous section on the spherical M5-brane at the vacuum, we study the effective Hamiltonian for the configuration of polarized M2-branes. We will see that the potential for the radius of this configuration coincides with (3.4) at large N. Consider an ansatz

$$Z^A = W^{\dagger A} = f(x)S^A, \qquad Z^\dagger_A = W_A = \bar{f}(x)S^\dagger_A, \ f(x) \in \mathbf{C},$$
$$A_\mu = a_\mu(x)S^B S^\dagger_B, \qquad \hat{A}_\mu = a_\mu(x)S^\dagger_B S^B, \ a_\mu(x) \in \mathbf{R}, \tag{4.1}$$

where S^A are the BPS matrices. This configuration represents the M2-branes polarized into a fuzzy S^3. The physical radius of the fuzzy S^3 is given by

$$R^2 = \frac{2(N-1)}{T_2}|f(x)|^2. \tag{4.2}$$

Under the ansatz, the covariant derivative becomes

$$D_\mu Z^A = (\partial_\mu f(x) + i a_\mu f(x)) S^A \equiv (\mathcal{D}_\mu f(x)) S^A \,, \tag{4.3}$$

while the gauge field strength satisfies

$$F_{\mu\nu} = (\partial_\mu a_\nu - \partial_\nu a_\mu) S^B S_B^\dagger \equiv f_{\mu\nu} S^B S_B^\dagger \,, \tag{4.4}$$

$$\hat{F}_{\mu\nu} = f_{\mu\nu} S_B^\dagger S^B \,. \tag{4.5}$$

The equations of motion for A_μ and $\hat{A}_\mu$ reduce to the following equation for a_μ,

$$\frac{k}{4\pi} \epsilon^{\rho\mu\nu} f_{\mu\nu} = 2i(f \mathcal{D}^\rho \bar{f} - \bar{f} \mathcal{D}^\rho f) \,. \tag{4.6}$$

For the case of the ansatz (4.1), we have

$$V_D = 0 \,, \qquad \frac{\partial \mathcal{W}}{\partial Z^A} = -\left(\frac{km}{8\pi} - \frac{1}{2}|f|^2 \right) \bar{f} S_A^\dagger \,, \qquad \frac{\partial \mathcal{W}}{\partial W_A} = -\left(\frac{km}{8\pi} - \frac{1}{2}|f|^2 \right) f S^A \,. \tag{4.7}$$

Therefore, from the equation (2.16), the effective Hamiltonian for the polarized M2-branes is given by

$$H = 2N(N-1) \int d^2x \left[\mathcal{D}_0 f \mathcal{D}_0 \bar{f} + \mathcal{D}_i f \mathcal{D}_i \bar{f} + \frac{64\pi^2}{k^2}|f|^2 \left(\frac{km}{8\pi} - \frac{1}{2}|f|^2 \right)^2 \right] \,. \tag{4.8}$$

This is nothing but the Hamiltonian for an abelian Chern-Simons Higgs model studied in [14, 18, 23] and [17]. Note that all the non-abelian structures in the model have been encoded into the overall factor $N(N-1)$ by the help of the BPS matrices. Although the gauge symmetry has been effectively reduced to the abelian symmetry, the Hamiltonian still describes N M2-branes.

It is known that the abelian Chern-Simons-Higgs model with the potential given in (4.8) exhibits a three-dimensional $\mathcal{N} = 2$ supersymmetry [15] and admits various class of BPS soliton solutions. See appendix C for the full structure of the Lagrangian including fermion parts. Following the analysis in [17], we find a Bogomol'nyi completion of the energy,

$$H = 2N(N-1) \int d^2x \left[\left| \mathcal{D}_0 f \pm if \left(m - \frac{4\pi}{k}|f|^2 \right) \cos\alpha \pm \mathcal{D}_2 f \sin\alpha \right|^2 \right.$$
$$\left. + \left| \mathcal{D}_1 f \mp i\mathcal{D}_2 f \cos\alpha \mp f \left(m - \frac{4\pi}{k}|f|^2 \right) \sin\alpha \right|^2 \right]$$
$$\pm \mathbf{B} \cos\alpha \mp \mathbf{S} \cos\alpha \mp \mathbf{P} \sin\alpha \pm \mathbf{T} \sin\alpha \,. \tag{4.9}$$

where we have used the gauge field equation of motion. Therefore we have an energy bound

$$E \geq 2N(N-1)\sqrt{(\mathbf{B} - \mathbf{S})^2 + (\mathbf{P} - \mathbf{T})^2}. \tag{4.10}$$

Here the following quantities have been defined,

$$\mathbf{B} \equiv \frac{km}{4\pi} \int d^2x \; B, \qquad B \equiv f_{12} \,, \tag{4.11}$$

$$\mathbf{S} \equiv i \int d^2x \; \left[\partial_1(f\mathcal{D}_2\bar{f}) - \partial_2(f\mathcal{D}_1\bar{f}) \right] , \tag{4.12}$$

$$\mathbf{P} \equiv \int d^2x \; \left[\mathcal{D}_0 f \mathcal{D}_2 \bar{f} + \mathcal{D}_2 f \mathcal{D}_0 \bar{f} \right] , \tag{4.13}$$

$$\mathbf{T} \equiv \int d^2x \; \mathcal{D}_1 \left(m|f|^2 - \frac{2\pi}{k}|f|^4 \right) . \tag{4.14}$$

Here $\mathbf{B}, \mathbf{S}, \mathbf{P}, \mathbf{T}$ are (time integrals of) the magnetic flux, the angular momentum, the linear momentum along x^2 direction, the tension of a domain wall. The energy bound (4.10) is saturated when the angle α satisfies the following condition

$$\cos\alpha = \frac{\mathbf{B} - \mathbf{S}}{\sqrt{(\mathbf{B} - \mathbf{S})^2 + (\mathbf{T} - \mathbf{P})^2}} \,, \qquad \sin\alpha = \frac{\mathbf{T} - \mathbf{P}}{\sqrt{(\mathbf{B} - \mathbf{S})^2 + (\mathbf{T} - \mathbf{P})^2}} \,. \tag{4.15}$$

The baryonic U(1) charge (2.11) for our ansatz is evaluated as

$$Q_b = 2iN(N-1) \int d^2x \; \left[f\mathcal{D}_0\bar{f} - \bar{f}\mathcal{D}_0 f \right] . \tag{4.16}$$

Using the equation of motion for the gauge field, we have the relation

$$Q_b = -\frac{k}{2\pi} N(N-1)\Phi \,, \qquad \Phi \equiv \int d^2x \; B \,. \tag{4.17}$$

The Noether charge is proportional to the magnetic flux. This is a specific property of Chern-Simons solitons. From the Bogomol'nyi completed form of the Hamiltonian (4.9), we obtain the BPS equations,

$$\mathcal{D}_0 f \pm if\left(m - \frac{4\pi}{k}|f|^2 \right)\cos\alpha \pm \mathcal{D}_2 f \sin\alpha = 0 \,, \tag{4.18}$$

$$\mathcal{D}_1 f \mp i\mathcal{D}_2 f \cos\alpha \mp f\left(m - \frac{4\pi}{k}|f|^2 \right)\sin\alpha = 0 \,. \tag{4.19}$$

Requiring that the supersymmetry transformations (2.22) and (2.23) of the fermionic fields vanish combined with these BPS equations yields the following conditions,

$$\epsilon_\alpha \pm \gamma^0_{\alpha\beta}\bar{\epsilon}^\beta \cos\alpha \pm i\gamma^1_{\alpha\beta}\bar{\epsilon}^\beta \sin\alpha = 0 \,, \tag{4.20}$$

$$\mp i\gamma^0_{\alpha\beta}\bar{\epsilon}^\beta \sin\alpha \mp \gamma^1_{\alpha\beta}\bar{\epsilon}^\beta \cos\alpha + i\gamma^2_{\alpha\beta}\bar{\epsilon}^\beta = 0 \,, \tag{4.21}$$

where $\gamma^\mu_{\alpha\beta}$ are the three-dimensional γ-matrices given in appendix B. This tells us that the BPS equations retain a quarter of the $\mathcal{N} = 2$ supersymmetry, specified by the effective supercharge (in the case $\cos\alpha = 1$)

$$\mathcal{Q} = Q_1 \pm iQ_2 + i\bar{Q}_1 \mp \bar{Q}_2 \,. \tag{4.22}$$

One should be careful about the above procedure deriving the BPS equations (4.18), (4.19). We first assumed an ansatz and derived the effective Hamiltonian (4.9) which is valid only for the special configuration (4.1). There is a possibility that the BPS equations are not consistent with the original equations of motion. Let us check this issue. Once the equations (4.18), (4.19) are satisfied, we have

$$\mathcal{D}_\mu \mathcal{D}^\mu f = m^2 f - \frac{16\pi m}{k}|f|^2 f + 3\left(\frac{16\pi^2}{k^2}\right)|f|^4 f.$$ (4.23)

This is the equation of motion (2.15) for $Z^A = W^{\dagger A}$ for the ansatz (4.1). Therefore the BPS equations (4.18), (4.19) are consistent with the full (second order) equations of motion for Z^A, W_A.

Note that in terms of the physical fuzzy S^3 radius R, the potential term in the equation (4.9) becomes

$$V(R) = \frac{4\pi^2 N T_2^3 R^2}{k^2(N-1)^2}\left(R^2 - \frac{(N-1)km}{2\pi T_2}\right)^2.$$ (4.24)

At large N, this gives

$$V(R) \sim T_5 \frac{T_2}{N}\left(R^3 - \frac{k}{4\pi}\frac{mN}{T_2}R\right)^2,$$ (4.25)

where we have used $T_p = (2\pi)^{-p} M_{11}^{p+1}$.

After a trivial rescaling of R, the result agrees with the potential (3.4) evaluated from the spherical M5-brane.[5] Once we consider the reducible ansatz $S^A \to \tilde{S}^A$, the potential is a sum of the each $S_{N_b}^A$ sectors, i.e., it is given by

$$V(R) \sim \sum_{b=1}^{l} T_5 \frac{T_2}{N_b}\left(R_b^3 - \frac{k}{4\pi}\frac{mN_b}{T_2}R_b\right)^2, \qquad \sum_{b=1}^{l} N_b = N.$$ (4.26)

This has the same form of the potential for the M2-branes distributed over a 3-sphere (3.7) giving a 3-sphere shells of M2-branes [8]. Therefore the reducible configuration can be interpreted as a set of N_b M2-branes polarized into fuzzy S^3 spheres with radii R_b. See figure 2 for a schematic picture.

A comment on this reducible configuration is in order. We have ignored off-block-diagonal parts of $\tilde{S}^A$ to derive the equation (4.26). The result is the sum of the potentials for the radii R_b inside N_b stack of M2-branes without any interactions among each stacks. Once we turn on the off-block-diagonal parts, it represents interactions among each stack of M2-branes and hence, in the dual picture, interactions among M5-branes.

---

[5]Strictly speaking, to compare the result with the dual supergravity side, we need to consider the reducible configuration (3.19) with two separated bunches of M2-branes. Assuming that the number of M2-branes in one of the bunches is larger than in the other one, the former can be considered as the source of the AdS_3 and the other can be identified with the probe M5-brane.
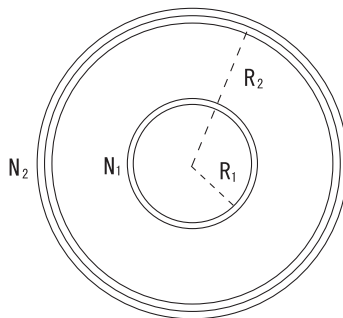
Figure 2. A schematic picture of the reducible fuzzy S^3 shell configuration for $l = 2$ case. A number of $N_1 = 2$, $N_2 = 3$ M2-branes are polarized around $R_1 \sim \sqrt{N_1}$ and $R_2 \sim \sqrt{N_2}$.

5 Solutions

In this section, we explore solutions for the BPS equations (4.18), (4.19). Since it is difficult to find solutions for general cases, we focus on the simplest situations. Let us first consider the $\cos \alpha = 1$ static case. In this case $(\mathbf{T} - \mathbf{P})$ must vanish and the energy bound is given by $\mathbf{B}$ and $\mathbf{S}$. The BPS equations reduce to

$$\mathcal{D}_1 f \mp i\mathcal{D}_2 f = 0 \,, \tag{5.1}$$

$$-\frac{k}{8\pi}B \pm |f|^2 \left(m - \frac{4\pi}{k}|f|^2 \right) = 0 \,. \tag{5.2}$$

This is nothing but the equation (14) in [18]. Following [14], we now assume an ansatz

$$f(r,\theta) = \lambda g(r)e^{in\theta} \,, \qquad \lambda^2 = \frac{km}{4\pi} \,, \qquad g(r) \in \mathbf{R} \,, \tag{5.3}$$

$$a_i(r) = \epsilon_{ij}\frac{\hat{x}^j}{r}[a(r) - n] \qquad (i = 1, 2) \,, \tag{5.4}$$

where (r, θ) are polar coordinates in the M2-brane world-volume and n is an integer. The BPS equations (5.1), (5.2) reduce to

$$g'(r) = \mp \frac{1}{r}a(r)g(r) \,, \tag{5.5}$$

$$\frac{a'(r)}{r} = \mp 2m^2 g^2(r)(g^2(r) - 1) \,. \tag{5.6}$$

Here the prime stands for the differentiation with respect to r. Requiring $a_i(r)$ and $g(r)$ to be non-singular at the origin, we obtain boundary conditions

$$a(0) - n = 0 \,, \qquad ng(0) = 0 \,. \tag{5.7}$$

On the other hand, the condition of finite energy is requiring that the scalar field settles down to its vacuum configuration at infinity, namely,

$$g(\infty) = \begin{cases} 0 & \text{(symmetric phase)} \,, \\ 1 & \text{(broken phase)} \,. \end{cases} \tag{5.8}$$

– 14 –

From the equations (5.5), (5.6), we obtain

$$(\ln g^2)'' + \frac{1}{r}(\ln g^2)' + 4m^2 g^2(1 - g^2) = 0. \tag{5.9}$$

When g is small, the $\mathcal{O}(g^4)$ term can be neglected and we can find an explicit solution [14]. That is given by

$$g(r) = \frac{\sqrt{8}s}{2mr}\left[\left(\frac{r}{r_0}\right)^s + \left(\frac{r_0}{r}\right)^s\right]^{-1}. \tag{5.10}$$

Here r_0, s are arbitrary constants. The energy density for the ansatz (5.3) and (5.4) is given by

$$E = \int_0^\infty dr \mathcal{E}, \tag{5.11}$$

$$\mathcal{E} = 4\pi r N(N-1)\lambda^2 \left((g')^2 + \frac{g^2 a^2}{r^2} + \left(\frac{a'}{2mrg}\right)^2 + m^2 g^2(g^2 - 1)^2\right). \tag{5.12}$$

The magnetic charges and angular momentum are also readily written down as

$$\Phi = \frac{km}{2}\int dr a'(r), \tag{5.13}$$

$$\mathbf{S} = 2\pi\lambda^2 \int dr (2a(x)g(x)g'(x) + a'(x)g^2(x)), \tag{5.14}$$

and $\mathbf{P} = 0$ and $\mathbf{T} = 0$. Considering the boundary conditions (5.7) and (5.8), (5.13) and (5.14) become

$$\Phi = -\frac{km}{2}(n + \alpha), \qquad \mathbf{S} = -2\pi\lambda^2\alpha\beta \tag{5.15}$$

where $\alpha = -a(\infty)$ and $\beta = g(\infty)$. In the following analysis, we consider the case $B < 0$, which corresponds to choosing the plus sign in (5.3) and (5.4). We will also see that $S = 0$ for all the cases we will discuss below. Because the detailed study of the global structure of these solutions has been performed in [14], we just briefly describe the solutions according to the boundary conditions.

5.1 Vortices

First consider the case of the boundary condition $g(\infty) = 1$. For the $n = 0$ case, the only solution is the vacuum configuration, $g(r) = 1$ and $a(r) = 0$. Thus it is just the spherical M5-brane discussed in section 3. For the $n \neq 0$ case, a topologically non-trivial configuration appears, which connects the boundaries $g(0) = 0, a(0) = n$ and $g(\infty) = 1, a(\infty) = 0$. They are vortex solutions. Although no explicit solution of the equation for this boundary condition is known, it is possible to solve it numerically. In figure 3, we plot $g(r)$ and $a(r)$ for the $n = 1$ solution and its corresponding energy density. We also show the magnetic flux density in figure 4. We can see that the topological charge is given by the magnetic flux $\Phi = -\frac{kmn}{2}$. As can be seen in figure 3, in almost all of the region of the M2-brane world-volume, the configuration represents the vacuum fuzzy S^3 discussed in section 3 but at the origin, there is a "dimple" inside which the full supersymmetry is recovered.
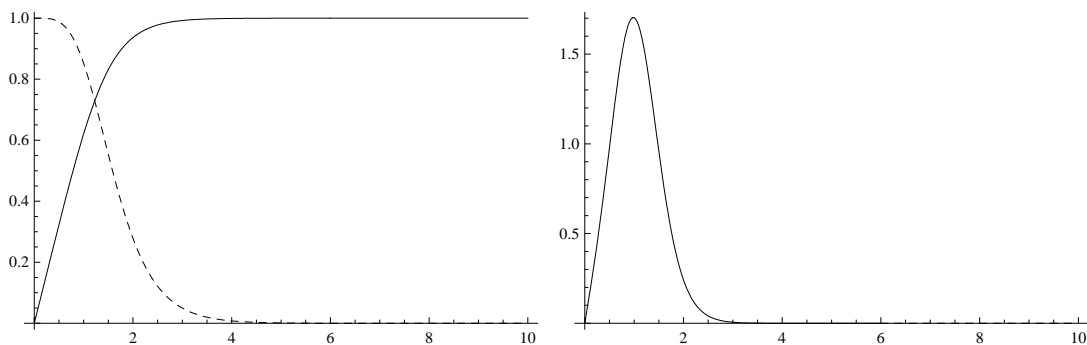
Figure 3. The left figure shows the profiles of the function $g(r)$ (solid line) and $a(r)$ (dashed line) for the topological vortex with $m = 1$. The right figure shows the behavior of corresponding energy density with $k = 1$ and $N = 2$.
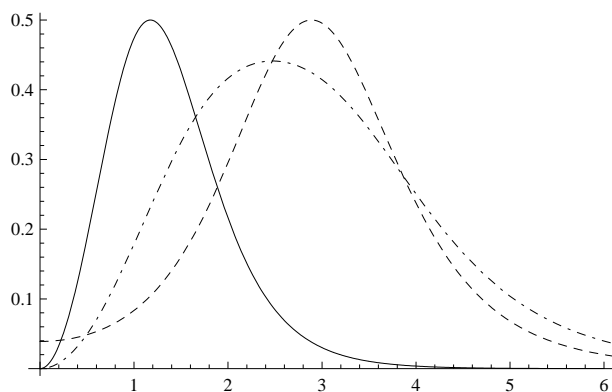

Figure 4. Plots of the absolute value of the magnetic flux density with $k = 1$ and $m = 1$. Solid, dashed, dot-dashed lines correspond to vortex, Q-ball and non-topological vortex, respectively.

5.2 Q-balls

Next we consider the case $g(\infty) = 0$ and $n = 0$. In this case, the scalar function $f(x)$ approaches the trivial vacuum and therefore all configurations are topologically trivial. The gauge field part $a(r)$ starts from the boundary $a(0) = 0$ and approaches some value $a(\infty)$ which characterizes the value of magnetic flux Φ. The magnetic flux is given by the U(1) Noether charge through the relation (4.17). This kind of solution is called a Q-ball. This is a lump-like object localized on a ring which surrounds the origin. Around the origin, the fuzzy S^3 has non-zero radius but the full supersymmetry is generically broken there. This is because the value of $g(r)$ at the origin need not to be the value in the vacuum. The situation is similar at infinity where the fuzzy sphere collapses into zero size but the gauge field has non-zero value. The plots of the solution and corresponding energy density are shown in figure 5. The behavior of the magnetic flux is shown in figure 4.
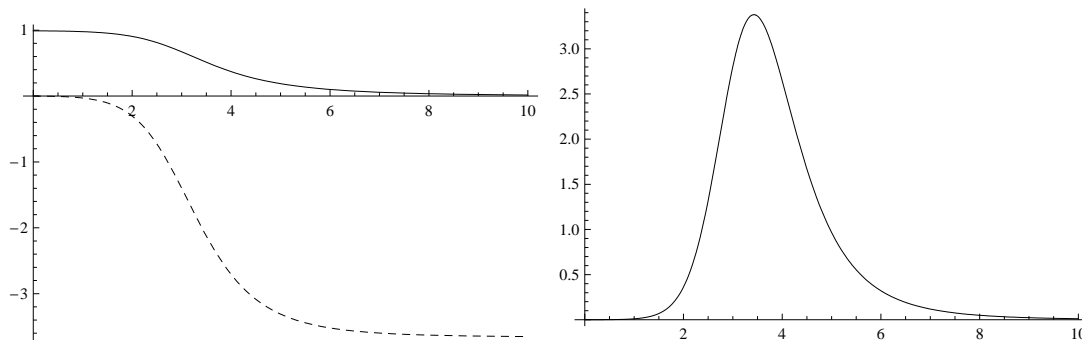
Figure 5. The left figure shows the profiles of the function $g(r)$ (solid line) and $a(r)$ (dashed line) for the Q-ball solution with $m = 1$. The right figure shows the behavior of corresponding energy density with $k = 1$ and $N = 2$.
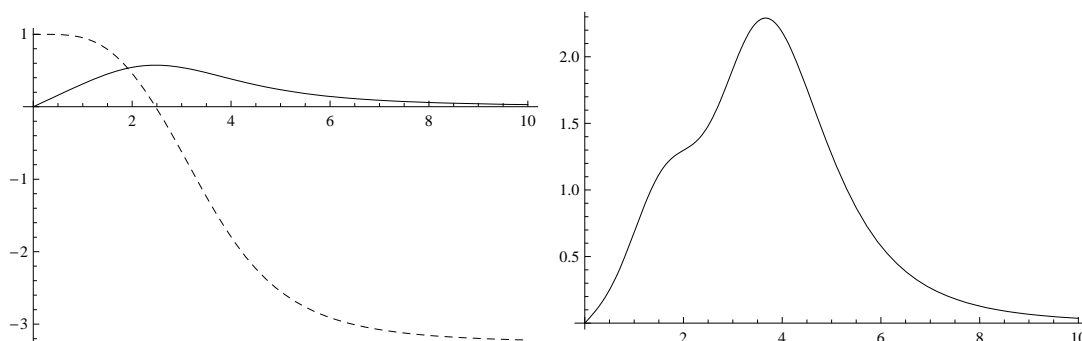

Figure 6. The left figure shows that the profiles of the function $g(r)$ (solid line) and $a(r)$ (dashed line) with $m = 1$. The right figure shows the behavior of corresponding energy density for the non-topological vortex with $k = 1$ and $N = 2$.

5.3 Non-topological vortices

Here we consider the case $g(\infty) = 0$ and $n \neq 0$. The solutions in this case are hybrids of the previous two cases. The large distance behaviors of $g(r)$ and $a(r)$ are the same as those in the Q-ball case, while they are similar to the vortex solution around the origin. These types of solutions are called non-topological vortices. Again, around the origin, the supersymmetry is recovered. We show the plots of this type of solution for the $n = 1$ case, its energy density in figure 6 and the flux density in figure 4.

5.4 Gauged domain wall

Next, let us consider $\cos \alpha = 1$ but non-static case. The BPS equations (4.18), (4.19) reduce to

$$\mathcal{D}_0 f \pm i f \left(m - \frac{4\pi}{k} |f|^2 \right) = 0 \,, \tag{5.16}$$

$$\mathcal{D}_1 f \mp i \mathcal{D}_2 f = 0 \,. \tag{5.17}$$

Considering the upper sign in the equations and assuming the following ansatz

$$f(t, x_1, x_2) = \phi(x_1)e^{-im(t+x_2)}, \qquad \phi(x_1) \in \mathbf{R}, \tag{5.18}$$

$$a_0(x_1) = a_2(x_1), \qquad a_1 = 0, \tag{5.19}$$

the equation (5.16) implies

$$a_0 = a_2 = \frac{4\pi}{k}\phi^2(x_1). \tag{5.20}$$

On the other hand, the equation (5.17) reduces to

$$\partial_1\phi - m\phi + \frac{4\pi}{k}\phi^3 = 0. \tag{5.21}$$

The gauge field equations of motion are satisfied provided ϕ satisfies the equation (5.21). Solutions of (5.21) are

$$\phi^2(x_1) = \frac{k}{4\pi}\frac{m}{1 \pm Ae^{-2mx_1}}, \tag{5.22}$$

where $A > 0$ is an integration constant. This solution was first found in [17] and was there called a "supertube" solution. The solution which has plus sign in front of $A = 1$ corresponds to a domain wall which interpolates between $\phi^2(-\infty) = 0$ and $\phi^2(+\infty) = \frac{km}{4\pi}$. At $x_1 = +\infty$, it describes an M5-brane wrapping an S^3 while at $x_1 = -\infty$, it is the trivial vacuum. Although we are considering the $\cos\alpha = 1$ case, and hence $\mathbf{T} - \mathbf{P} = 0$, each $\mathbf{T}$ and $\mathbf{P}$ have non-vanishing values. Indeed, the tension and the momentum for the solution are the same value $\mathbf{T} = \mathbf{P} = km2/8\pi2$ and $\mathbf{S} = 0$. This fact means that the domain wall tension is canceled by the momentum and makes it bend freely [17].

This solution would be interpreted as a brane configuration by following the discussion in section 3. In the solution, $\phi(x_1)$ part corresponds to the 1/2 BPS domain wall solution (3.16). An M5-brane at $x_1 > 0$ bends and an M5-brane filling B^4 may appear at the origin. On the other hand, the phase factor $e^{-im(t+x_2)}$ acts as a rotation in the 3-7, 4-8, 5-9, 6-10 plane [8]. This can be seen by decomposing $Z^A = X^{A+2} + iX^{A+6}$, $W^{\dagger A} = X^{A+4} + iX^{A+8}$ where the X and iX parts correspond to hermitian and anti-hermitian parts of the bi-fundamental scalar fields. Therefore, the solution can be interpreted as a configuration of the bent branes rotating in time and its angle also depends on x_2.

6 Conclusions and discussions

In this paper, we investigated the ABJM model deformed by an F-term mass. The mass term generically breaks $\mathcal{N} = 6$ maximal supersymmetry down to $\mathcal{N} = 2$. A vacuum of the model corresponding to an irreducible representation is found to be a fuzzy S^3 representing an M5-brane with topology $\mathbf{R}^{1,2} \times S^3$ at large-N. We find that the radius of the sphere coincides with the one found in [8]. The tension of the domain wall solutions is evaluated and compared with the result of the dual M5-brane calculation. With this observation, we identified the F-term mass m as the VEV of the anti-self-dual 4-form flux

T_4 in the eleven-dimensional supergravity that was introduced in [8]. Therefore the vacuum configuration can be interpreted as Myers' dielectric effect making N M2-branes puff up into a fuzzy sphere.

In the latter half of this paper, we assumed a polarized M2-brane configuration. We showed that the mass deformed ABJM model reduces to the abelian Chern-Simons-Higgs model with 6th power polynomial potential studied in detail by Jackiw-Lee-Weinberg [14]. The potential part of this effective model is just the potential for the radius of the spherical M5-brane. This effective model admits not only BPS topological vortices and domain walls but also non-topological vortices and Q-balls. We find that these BPS topological/non-topological solitons preserve $1/4$ of the manifest $\mathcal{N} = 2$ supersymmetry. Because these solutions can exist only in the $m \neq 0$ case, these configurations are supported against collapse by the non-zero background flux T_4. The situation is quite similar to the massive BLG model where various BPS soliton solutions are possible [24, 25].

A comment on the number of supersymmetries is in order. In the massless ABJM model, $SU(2) \times SU(2)$ global symmetry together with $SU(2)_R$ symmetry gives $SU(4)_R \sim SO(6)_R$ implying the existence of $\mathcal{N} = 6$ supersymmetry. Indeed, in [4], the explicit on-shell $\mathcal{N} = 6$ supersymmetry transformation of component fields was found. Our case is similar to the massless ABJM model. The global $SU(2) \times SU(2)$ symmetry is broken down to $SU(2)_{\text{diag}}$ by the mass term. However, this $SU(2)_{\text{diag}}$ does not commute with the remaining $SU(2)_R$. These two $SU(2)$s are combined into an $SO(4)_R$ which would imply an enhancement of $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 4$. Presumably, this supersymmetry is enhanced to the maximal one since our deformation term (2.14) (together with its conjugate term) does not break the $U(1)_R$ and $\mathbf{Z}_2$ symmetries and these are combined with the $SU(2)$s giving $SU(2) \times SU(2) \times U(1)_R \times \mathbf{Z}_2$ symmetry as discussed in [12]. Although we do not present it here, it would be possible to explicitly find the enhanced supersymmetries by the Noether method or requiring that the supersymmetry algebra closes.

Another important issue is the understanding of the solution in the dual spherical M5-brane world-volume picture. Because our effective potential is identified with the radius potential of the spherical M5-brane in the presence of the anti-self-dual 4-form flux, the BPS solutions discussed in this paper correspond to BPS configurations of this spherical M5-brane. It seems possible to find these BPS configurations by promoting the constant radius r in (3.4) to a "field" $r = r(x_1, x_2)$ on the M2-brane world-volume and solving the BPS equations obtained from the promoted spherical M5-brane action. This issue is beyond the scope of this paper and we leave it for future work.

Finally, let us comment on the dimensional reduction of the ABJM model. Since the ABJM model describes N coincident M2-branes in eleven dimensions, once we compactify one direction along the M2-brane world-volume, the model should describe the low energy effective theory of N coincident fundamental strings (F1) in type IIA string theory. This theory is similar to the matrix string theory [26] representing a polarization of the F1 due to the background flux. The configuration corresponding to the M2-branes polarized into the M5-brane now reduces to fundamental strings polarized into a D4-brane.[6] The anti-

---

[6]A similar configuration for F1 polarized into D2 was studied in the dilute set of D0-branes having the

self-dual 4-form flux in eleven-dimensions is reduced to the anti-self-dual RR 4-form flux in ten-dimensions. The multiple F1 is polarized into a fuzzy S^3 due to this flux. Since the potential terms both in the M5-brane action and the massive ABJM model do not change under the dimensional reduction, the discussion in section 3 holds even for the D4/F1 case implying AdS_3/CFT_2 duality.

On the other hand, the effective Lagrangian for the polarized M2-branes discussed in section 4 reduces to the one for the polarized F1. One component of the gauge field a_μ becomes an auxiliary field after the dimensional reduction. The resultant theory is a new two dimensional BF-like theory. We find that this model is exactly the one studied in [28]. It was shown that the model exhibits $\mathcal{N} = (2,2)$ supersymmetry in two-dimensions and have a BPS domain wall solution. It would be interesting to examine this solution in terms of brane configurations in ten-dimensions.

Acknowledgments

A BPS matrices

The explicit form of the BPS matrices was first constructed in [12] to find a vacuum of the mass deformed ABJM model. The BPS matrices satisfy the following conditions,

$$S^A = S^B S_B^\dagger S^A - S^A S_B^\dagger S^B \,, \tag{A.1}$$
$$S_A^\dagger = S_A^\dagger S^B S_B^\dagger - S_B^\dagger S^B S_A^\dagger \,. \tag{A.2}$$

The explicit form of the matrices satisfying the above conditions is given by

$$(S_1^\dagger)_{mn} = \sqrt{m-1}\,\delta_{mn}\,, \qquad (S_2^\dagger)_{mn} = \sqrt{N-m}\,\delta_{m+1,n}\,, \tag{A.3}$$
$$S^1 S_1^\dagger = \mathrm{diag}(0,1,2,\ldots,N-1) = S_1^\dagger S^1 \,, \tag{A.4}$$
$$S_2^\dagger S^2 = \mathrm{diag}(N-1,N-2,\ldots,1,0)\,, \tag{A.5}$$
$$S^2 S_2^\dagger = \mathrm{diag}(0,N-1,N-2,\ldots,1)\,, \tag{A.6}$$
$$S_A^\dagger S^A = (N-1)\mathbf{1}_{N\times N}\,. \tag{A.7}$$

Therefore we have

$$\mathrm{Tr}S^A S_A^\dagger = \mathrm{Tr}S_A^\dagger S^A = N(N-1). \tag{A.8}$$

___
F1 charge [27].

These matrices satisfy the following relations,

$$\epsilon_{CD}S^C S_A^\dagger S^D = \epsilon_{AB}S^B\,, \qquad \epsilon^{CD}S_C^\dagger S^A S_D^\dagger = -\epsilon^{AB}S_B^\dagger\,, \tag{A.9}$$

$$\epsilon_{AC}\epsilon^{CB} = \delta_A{}^B\,, \qquad\qquad \epsilon^{AC}\epsilon_{CB} = \delta^A{}_B\,, \qquad \epsilon^{12} = -\epsilon_{12} = 1\,. \tag{A.10}$$

B $\mathcal{N} = 2$ superfield formulation of ABJM model

Here we briefly review the $\mathcal{N} = 2$ superfield formulation of the mass-deformed ABJM model given in section 2. We follow the same notation as in [19]. In terms of $\mathcal{N} = 2$ superfields, the mass-deformed ABJM model is described by chiral superfields $\mathcal{Z}^A, \mathcal{W}_A, \bar{\mathcal{Z}}_A, \bar{\mathcal{W}}^A (A = 1, 2)$ and two vector superfields $\mathcal{V}, \hat{\mathcal{V}}$. The chiral superfields $\mathcal{Z}^A, \mathcal{W}_A (A = 1, 2)$ and $\bar{\mathcal{Z}}_A, \bar{\mathcal{W}}^A$ transform in the bi-fundamental representations $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$ under $\mathrm{U}(N) \times \mathrm{U}(N)$, respectively. They are expanded in components

$$\mathcal{Z}(x_L) = Z(x_L) + \sqrt{2}\theta\zeta(x_L) + \theta^2 F(x_L)\,, \tag{B.1}$$

$$\bar{\mathcal{Z}}(x_R) = Z^\dagger(x_R) - \sqrt{2}\bar{\theta}\zeta^\dagger(x_R) - \bar{\theta}^2 F^\dagger(x_R)\,, \tag{B.2}$$

$$\mathcal{W}(x_L) = W(x_L) + \sqrt{2}\theta\omega(x_L) + \theta^2 G(x_L)\,, \tag{B.3}$$

$$\bar{\mathcal{W}}(x_R) = W^\dagger(x_R) - \sqrt{2}\bar{\theta}\omega^\dagger(x_R) - \bar{\theta}^2 G^\dagger(x_R)\,. \tag{B.4}$$

Here $x_L^\mu = x^\mu + i(\theta\gamma^\mu\bar{\theta})$ and $x_R^\mu = x^\mu - i(\theta\gamma^\mu\bar{\theta})$, where $\gamma_{\alpha\beta}^\mu$ are the three-dimensional γ-matrices defined by $\gamma_{\alpha\beta}^\mu = (-\mathbf{1}, -\sigma^3, \sigma^1)$. The vector superfields $\mathcal{V}$ and $\hat{\mathcal{V}}$ transform in the adjoint representations $(\mathbf{adj}, 1)$ and $(1, \mathbf{adj})$ of $\mathrm{U}(N) \times \mathrm{U}(N)$. The component expansion in the Wess-Zumino gauge is given by

$$\mathcal{V}(x) = 2i\theta\bar{\theta}\sigma + 2\theta\gamma^\mu\bar{\theta}A_\mu + \sqrt{2}i\bar{\theta}^2\theta\chi - \sqrt{2}i\theta^2\bar{\theta}\bar{\chi} + \theta^2\bar{\theta}^2 D\,, \tag{B.5}$$

and similarly for $\hat{\mathcal{V}}$. The action in terms of $\mathcal{N} = 2$ superfield is written by

$$S = S_{\mathrm{kin}} + S_{\mathrm{CS}} + S_{\mathrm{sp}}\,, \tag{B.6}$$

where

$$S_{\mathrm{kin}} = \int d^3x d^4\theta \mathrm{Tr}\left(-\bar{\mathcal{Z}}_A e^{-\mathcal{V}}\mathcal{Z}^A e^{-\hat{\mathcal{V}}} - \bar{\mathcal{W}}^A e^{-\hat{\mathcal{V}}}\mathcal{W}_A e^{\mathcal{V}}\right)\,, \tag{B.7}$$

$$S_{\mathrm{CS}} = -\frac{ik}{8\pi}\int d^3x d^4\theta \int_0^1 dt \mathrm{Tr}\left(\mathcal{V}\bar{D}^\alpha(e^{t\mathcal{V}}D_\alpha e^{-t\mathcal{V}}) - \hat{\mathcal{V}}\bar{D}^\alpha(e^{t\hat{\mathcal{V}}}D_\alpha e^{-t\hat{\mathcal{V}}})\right)\,, \tag{B.8}$$

$$S_{\mathrm{sp}} = \frac{8\pi}{k}\left(\int d^3x d^2\theta W(\mathcal{Z}, \mathcal{W}) + \int d^3x d^2\bar{\theta}\bar{\mathcal{W}}(\bar{\mathcal{Z}}, \bar{\mathcal{W}})\right)\,, \tag{B.9}$$

where the superpotential is given by (2.12), (2.13) and (2.14). Substituting the expressions (B.1)–(B.5) into the action and integrating over the Grassmann measure one obtains the off-shell component action. Further elimination of the set of auxiliary fields yields the

on-shell component action. The equations of motion are given by

$$F^A = \frac{2\pi}{k}\left(2\epsilon^{AC}\epsilon_{BD}W^{\dagger B}Z^\dagger_C W^{\dagger D} - \frac{km}{2\pi}W^{\dagger A}\right),\tag{B.10}$$

$$G_A = \frac{2\pi}{k}\left(-2\epsilon_{AC}\epsilon^{BD}Z^\dagger_B W^{\dagger C}Z^\dagger_D - \frac{km}{2\pi}Z^\dagger_A\right),\tag{B.11}$$

$$N^A = \sigma Z^A - Z^A\hat{\sigma}, \qquad M_A = \hat{\sigma}W_A - W_A\sigma,\tag{B.12}$$

$$\hat{\sigma}^a = \frac{2\pi}{k}\text{Tr}T^a(ZZ^\dagger - W^\dagger W), \qquad \hat{\sigma}^a = \frac{2\pi}{k}\text{Tr}T^a(Z^\dagger Z - WW^\dagger),\tag{B.13}$$

$$\chi^a = -\frac{4\pi}{k}\text{Tr}T^a(Z\zeta^\dagger - \omega^\dagger W), \qquad \hat{\chi}^a = -\frac{4\pi}{k}\text{Tr}T^a(\zeta^\dagger Z - W\omega^\dagger).\tag{B.14}$$

The bosonic on-shell component action is described in (2.1), (2.2), (2.3) and (2.4).

C $\mathcal{N} = 2$ abelian Chern-Simons-Higgs model from ABJM

In this appendix, we see the full structure of the effective Lagrangian on polarized M2-branes. In addition to the ansatz for the bosonic parts (4.1), we consider the following fermionic ansatz,

$$\zeta^A = -i\omega^{\dagger A} \equiv \psi S^A,\tag{C.1}$$

where ψ_α is a complex two component spinor. Then the effective Lagrangian corresponding to the ansatz (4.1), (C.1) is

$$\mathcal{L}_{\text{eff}} = 2N(N-1)\left[\frac{k}{16\pi}\epsilon^{\mu\nu\rho}a_\mu f_{\nu\rho} - |\mathcal{D}_\mu f|^2 - i\psi^\dagger\gamma^\mu\mathcal{D}_\mu\psi\right.$$
$$\left. - \frac{16\pi^2}{k^2}|f|^2\left(|f|^2 - \frac{km}{4\pi}\right)^2 + i\left(\frac{4\pi}{k}\right)\left(3|f|^2 - \frac{km}{4\pi}\right)\psi^\dagger\psi\right].\tag{C.2}$$

This result precisely gives $\mathcal{N} = 2$ supersymmetric abelian Chern-Simons-Higgs model studied in [15] after suitable rescaling of variables. The $\mathcal{N} = 2$ supercharge is given by

$$q = \int d^2x\left[\gamma^\mu\gamma^0\psi\mathcal{D}_\mu\bar{f} - \frac{k}{4\pi}\gamma^0\psi\bar{f}\left(|f|^2 - \frac{km}{4\pi}\right)\right].\tag{C.3}$$

This satisfies the relation

$$\{q_\alpha, \bar{q}^\beta\} = -i(\gamma^\mu)_\alpha{}^\beta P_\mu - \delta_\alpha{}^\beta Z.\tag{C.4}$$

Here P_μ is the momentum operator and Z is the central charge given by

$$Z = \int d^2x\left[-\frac{1}{2}\varepsilon^{ij}f_{ij}|f|^2 + \frac{4\pi}{k}i\left(|f|^2 - \frac{km}{4\pi}\right)(\bar{f}\mathcal{D}_0 f - f\mathcal{D}_0\bar{f} - i\bar{\psi}\gamma^0\psi)\right],\tag{C.5}$$

where $\varepsilon^{12} = -1$. The central charge corresponding to the BPS configurations in section 5 is easily evaluated. The result is

$$Z = \frac{km}{4\pi}\int d^2x\; B.\tag{C.6}$$

This correctly reproduces the energy bound discussed in section 5.

References

[1] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D 75** (2007) 045020 [hep-th/0611108] [SPIRES]; *Gauge Symmetry and Supersymmetry of Multiple M2-Branes*, *Phys. Rev.* **D 77** (2008) 065008 [arXiv:0711.0955] [SPIRES]; *Comments On Multiple M2-branes*, *JHEP* **02** (2008) 105 [arXiv:0712.3738] [SPIRES].

[2] A. Gustavsson, *Algebraic structures on parallel M2-branes*, *Nucl. Phys.* **B 811** (2009) 66 [arXiv:0709.1260] [SPIRES].

[3] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

[4] S. Terashima, *On M5-branes in $N = 6$ Membrane Action*, *JHEP* **08** (2008) 080 [arXiv:0807.0197] [SPIRES].

[5] C. Krishnan and C. Maccaferri, *Membranes on Calibrations*, *JHEP* **07** (2008) 005 [arXiv:0805.3125] [SPIRES].

[6] K. Hanaki and H. Lin, *M2-M5 Systems in $N = 6$ Chern-Simons Theory*, *JHEP* **09** (2008) 067 [arXiv:0807.2074] [SPIRES].

[7] T. Fujimori, K. Iwasaki, Y. Kobayashi and S. Sasaki, *Time-dependent and Non-BPS Solutions in $N = 6$ Superconformal Chern-Simons Theory*, *JHEP* **12** (2008) 023 [arXiv:0809.4778] [SPIRES].

[8] I. Bena, *The M-theory dual of a 3 dimensional theory with reduced supersymmetry*, *Phys. Rev.* **D 62** (2000) 126006 [hep-th/0004142] [SPIRES].

[9] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136 [SPIRES].

[10] I. Bena and N.P. Warner, *A harmonic family of dielectric flow solutions with maximal supersymmetry*, *JHEP* **12** (2004) 021 [hep-th/0406145] [SPIRES].

[11] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174] [SPIRES].

[12] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, *A Massive Study of M2-brane Proposals*, *JHEP* **09** (2008) 113 [arXiv:0807.1074] [SPIRES].

[13] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053] [SPIRES].

[14] R. Jackiw, K.-M. Lee and E.J. Weinberg, *Selfdual Chern-Simons solitons*, *Phys. Rev.* **D 42** (1990) 3488 [SPIRES].

[15] C.-k. Lee, K.-M. Lee and E.J. Weinberg, *Supersymmetry and selfdual Chern-Simons systems*, *Phys. Lett.* **B 243** (1990) 105 [SPIRES].

[16] T.D. Lee and Y. Pang, *Nontopological solitons*, *Phys. Rept.* **221** (1992) 251 [SPIRES].

[17] S. Kim, K.-M. Lee and H.-U. Yee, *Supertubes in field theories*, *Phys. Rev.* **D 75** (2007) 125011 [hep-th/0603179] [SPIRES].

[18] J. Hong, Y. Kim and P.Y. Pac, *On the multivortex solutions of the abelian Chern-Simons-Higgs theory*, *Phys. Rev. Lett.* **64** (1990) 2230 [SPIRES].

[19] M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons Theories and AdS_4/CFT_3 Correspondence*, *JHEP* **09** (2008) 072 [arXiv:0806.1519] [SPIRES].

[20] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds*, *JHEP* **09** (2008) 002 [arXiv:0806.4977] [SPIRES].

[21] P. Pasti, D.P. Sorokin and M. Tonin, *Covariant action for a D = 11 five-brane with the chiral field*, *Phys. Lett.* **B 398** (1997) 41 [hep-th/9701037] [SPIRES].

[22] M. Perry and J.H. Schwarz, *Interacting chiral gauge fields in six dimensions and Born-Infeld theory*, *Nucl. Phys.* **B 489** (1997) 47 [hep-th/9611065] [SPIRES].

[23] R. Jackiw and E.J. Weinberg, *Selfdual Chern-Simons vortices*, *Phys. Rev. Lett.* **64** (1990) 2234 [SPIRES].

[24] K. Hosomichi, K.-M. Lee and S. Lee, *Mass-Deformed Bagger-Lambert Theory and its BPS Objects*, *Phys. Rev.* **D 78** (2008) 066015 [arXiv:0804.2519] [SPIRES].

[25] J. KIm and B.-H. Lee, *Abelian Vortex in Bagger-Lambert-Gustavsson Theory*, *JHEP* **01** (2009) 001 [arXiv:0810.3091] [SPIRES].

[26] R. Dijkgraaf, E.P. Verlinde and H.L. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030] [SPIRES].

[27] I. Bena, *The polarization of F1 strings into D2 branes: "Aut Caesar aut nihil"*, *Phys. Rev.* **D 67** (2003) 026004 [hep-th/0111156] [SPIRES].

[28] H.-c. Kao, K.-M. Lee and T. Lee, *The BPS domain wall solutions in self-dual Chern-Simons-Higgs systems*, *Phys. Rev.* **D 55** (1997) 6447 [hep-th/9612183] [SPIRES].